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HCL-003-001513

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October - 2017

Mathematics : BSMT - 501 (A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003

Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 Answer the following questions in short : **20**

(1) Define Open set

(2) Give an example of a subset of metric space R which is not open and closed

(3) Define Isolated point

(4) Define Derived set

(5) If $E=[1,2]$ is a subset of metric space R then find E^0

(6) Define Lower Riemann Integration

(7) If $f(x)=\frac{20}{x}, x \in [2,20]$ and partition $P=\{2,4,5,20\}$ then find $\|P\|$

(8) State fundamental theorem of Riemann Integration

(9) Define Riemann sum

(10) State General form of first mean value theorem

(11) Define Right coset

(12) Write the order of the set A_n of all even permutations $S_n(n \geq 2)$

(13) Define Permutation

(14) If $f=(1,2,3)(4,6,5,7,8)$ then find $O(f)$, where $f \in S_8$

(15) For group $(Z_5,+_5), O(4) =$ ________

(16) Define Inner automorphism

(17) For the group $(Z_6,+_6)$ find generators of Z_6

(18) Define Index of subgroup H in group G

(19) Define Normal subgroup

(20) Is S_3 and Z_6 are isomorphic? Give the reason.

2 (A) Answer any **three** : **6**

(1) Define Norm of partition and finer partition.

(2) If (X,d) is a metric space and $A,B \subset X$ and $A \subset B$ then $\overline{A} \subset \overline{B}$

(3) Obtain border set of the subset $(-1,1)$ of metric space R.

(4) Determine whether set $[0,\infty)$ of metric space R is open or closed set.

(5) If $f(x)=\frac{20}{x}, x \in [2,20]$ and partition $P=\{2,4,5,20\}$ then find $U(P,f)$

(6) Evaluate: $\lim_{n\to\infty} n\left[\frac{1}{n^2+1^2}+\frac{1}{n^2+2^2}+....+\frac{1}{2n^2}\right]$

(B) Answer any **three** : **9**

(1) Prove that every continuous function f on [a,b] is Riemann integrable on [a,b]

(2) If f is decreasing function 'on [a,b] then prove that f is R-integrable.

(3) If (X,d) is a metric space and $A,B \subset X$ then prove that $(A \cup B)' = A' \cup B'$

(4) If f and g are R-integrable on [a,b] then prove that $f+g$ is also R-integr-able on [a,b]

(5) Prove that every finite subset of any metric space is a closed set.

(6) Prove that union of finite closed sets of metric space is a closed set:

(C) Answer any **two** : **10**

(1) In usual notations prove that $\overline{E}$ is a closed set in metric space.

(2) State. and prove general form of first mean value theorem.

(3) Prove that $\frac{3}{4}$ is in cantor set.

(4) Prove that $\frac{\pi^2}{10} \le \int_0^{\pi} \frac{x}{3 - 2\cos x} dx \le \frac{\pi^2}{2}$

(5) Prove that $\lim_{n \to \infty} \left(\frac{n^n}{n!} \right)^{\frac{1}{n}} = e$

3 (A) Answer any **three** : **6**

(1) If $\sigma = (1\ 2\ 3\ 4), \sigma \in S_5$ then find σ^{-1}

(2) If $f : R \to R$ is defined as $f(x) = x^2$ then check whether f is a permutation or not.

(3) Define: Index of a subgroup in the group.

(4) Prove that the inverse element is unique in the group.

(5) If $a^2 = e$ for each element a of a group G then show that G is commutative.

(6) Check whether $(Z, +)$ is cyclic group or not.

(B) Answer any **three** : **9**

(1) Let $H \le G$ and let $a, b \in G$ then show that $aH = Ha \Leftrightarrow ab^{-1} \in H$

(2) Prove that intersection of two subgroups of a group is also a subgroup.

(3) State Lagrange's theorem. Converse of this is true? If not then give example

(4) Draw the lattice diagram of Z_8

(5) If H and K are any two subgroups of group G such that $\left(O\left(H, O\left(K\right)\right)=1\right.$ then show $H \cap K = \{e\}$

(6) If H is a normal subgroup of group G with $i_G\left(H\right)=m$ then prove that $a^m \in H; \forall\, a \in G$

(C) Answer any **two** : **10**

(1) State and prove Lagrange's theorem. for finite groups

(2) Prove that a group cannot be a union of its two proper subgroups.

(3) State and. prove Cayley's theorem.

(4) Prove that the combination of two disjoint cycles in S_n is commutative.

(5) Show that $\left(R,+\right) \cong \left(R_+,\cdot\right)$